[9] J. L. Jackel, V. Ramaswamy, and S. Lyman, "Elimination of out-diffused surface guiding in Ti-diffused $LiNbO_3$ waveguides," *Appl. Phys. Lett.*, vol. 38, pp. 509–511, 1981.
[10] V. Ramaswamy, R. C. Alferness, and M. D. Divino, "High efficiency single-mode fiber to Ti:$LiNbO_3$ waveguide coupling," *Electron. Lett.*, vol. 10, pp. 30–31, 1982.
[11] W. K. Burns and G. B. Hocker, "End fire coupling between optical fibers and diffused channel waveguides," *Appl. Opt.*, vol. 16, pp. 2048–2049, 1977.
[12] M. Minakata, S. Saito, M. Shibata, and S. Miyazawa, "Precise determination of refractive-index changes in Ti-diffused $LiNbO_3$ waveguides," *J. Appl. Phys.*, vol. 49, pp. 4677–4682, 1978.
[13] S. K. Korotky, W. J. Minford, L. L. Buhl, M. D. Divino, and R. C. Alferness, "Mode size and method for estimating the propagation constant of single-mode Ti:$LiNbO_3$ strip waveguides," this issue, pp. 1784–1789.
[14] D. Marcuse, to be published.
[15] R. C. Alferness, L. L. Buhl, and M. D. Divino, "Low-loss fiber-coupled directional coupler modulator," to be published.
[16] I. P. Kaminow, private communication.

Rod C. Alferness, for a photograph and biography, see this issue, p. 1789.

V. R. Ramaswamy (M'62–SM'80), photograph and biography not available at the time of publication.

Steven K. Korotky, for a photograph and biography, see this issue, p. 1789.

Manuel D. Divino, for a photograph and biography, see this issue, p. 1789.

Lawrence L. Buhl, for a photograph and biography, see this issue, p. 1789.

Analysis of Integrated-Optics Near 3 dB Coupler and Mach–Zehnder Interferometric Modulator Using Four-Port Scattering Matrix

ROBERT H. REDIKER, FELLOW, IEEE, AND FREDERICK J. LEONBERGER, SENIOR MEMBER, IEEE

Abstract—The scattering matrix formalism for a lossless four-port device is used to describe the interferometric performance of the integrated-optics near 3 dB coupler and, consequently, the Mach–Zehnder interferometric modulator as a function of coupler and/or power imbalance. For the case of a coupler consisting of three single-mode dielectric guides forming a Y junction, a fourth port is incorporated which takes all the power radiated out of the guided-wave system in the vicinity of the junction. The interferometric properties of the coupler are shown to be relatively insensitive to fabrication and/or design errors of a magnitude which would make the use of this junction in the reverse direction as a 3 dB divider very marginal. A coupler with an extinction ratio as an interferometer better than −26 dB corresponds to a power divider which couples 22 percent more power into one arm than the other. It is also shown that the near 3 dB coupler used as the output of an interferometric modulator is similarly insensitive to the inequality of the powers in the two arms.

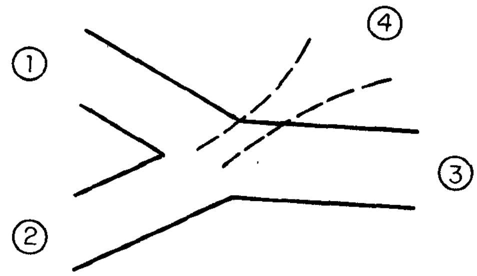


Fig. 1. Four-port representation of the 3 dB coupler. The coupler is exemplified by a Y junction of single-mode dielectric waveguides. The fourth port takes all the power radiated out of the guided-wave system in the vicinity of the Y junction for input power at port 1 and/or port 2.

I. Introduction

In this paper the scattering matrix formalism for a lossless four-port device which has been used successfully in the analysis of microwave circuits [1] is used to describe the interferometric performance of the integrated-optics near 3 dB coupler. A near 3 dB coupler is defined here as a real achievement of an ideal 3 dB coupler for which there is an imbalance in the power splitting. The near 3 dB coupler in integrated optics could be formed as a directional coupler or as a Y junction. Here we will take an example, a Y junction that consists in the forward direction of two single-mode dielectric waveguides coming together into a third single-mode guide, as shown in Fig. 1. Also, as shown in the figure, we include at the junction a fourth port which takes all the power radiated out of the guided-wave system in the vicinity of the coupler. Inclusion of this fourth port allows us to analyze this coupler as a lossless four-port network. The properties of the scattering matrix of the lossless four-port network—the matrix is

Manuscript received April 1, 1982; revised June 7, 1982. This work was supported by the Department of the U.S. Air Force.
The authors are with Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, MA 02173.

0018-9480/82/1000-1801$00.75 © 1982 IEEE

both symmetrical and unitary—are used to an advantage to obtain unknown matrix coefficients from those that are *a priori* given, and thus, to be able to completely determine the outputs from the Y junction for arbitrary combinations of inputs. In particular, the power in the output, port 3 in Fig. 1, is obtained as a function of the respective powers and their relative phase in the two inputs, ports 1 and 2, under the assumption that the device of Fig. 1 in the backward direction acts as a known near 3 dB splitter.

In determining the scattering matrix for the four port, it is assumed that there is no coupling between the waves traveling toward the coupler junction ($+z$ waves) and those traveling away from the junction ($-z$ waves), either between the waves associated with port 1 and those associated with port 2 or vice-versa, or between waves associated with an individual port. This assumption is equivalent to neglecting the reflected waves and is appropriate for the ***integrated-optics dielectric-waveguide junctions***. It follows from this assumption and the symmetry requirements of the scattering matrix that, for the dielectric waveguide Y junction, there is no power coupled to radiation modes into the crystal when the coupler is operated as a power divider (i.e., there is no power coupled from port 3 to port 4). The zero coupling to radiation modes for dielectric waveguide Y junctions operated as a power divider is commonly used in conventional treatments of these junctions.

With the assumption of no reflected waves and that the near 3 dB coupler is characterized by the coupling of $(1/2)(1+\delta)$ of power from port 3 into port 1 and $(1/2)(1-\delta)$ of the power into port 2, we can determine the entire four-port scattering matrix between the output electric fields $\boldsymbol{E}'$ and input fields $\boldsymbol{E}$. This matrix is

$$\begin{bmatrix} E_1' \\ E_2' \\ E_3' \\ E_4' \end{bmatrix} = \begin{bmatrix} 0 & 0 & \sqrt{\frac{1+\delta}{2}} & \sqrt{\frac{1-\delta}{2}} \\ 0 & 0 & \sqrt{\frac{1-\delta}{2}} & -\sqrt{\frac{1+\delta}{2}} \\ \sqrt{\frac{1+\delta}{2}} & \sqrt{\frac{1-\delta}{2}} & 0 & 0 \\ \sqrt{\frac{1-\delta}{2}} & -\sqrt{\frac{1+\delta}{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}. \tag{1}$$

To obtain this matrix, we have made use of our knowledge that it must be unitary and symmetrical.

From the matrix of (1)

$$E_3' e^{i\theta} = \sqrt{\frac{1+\delta}{2}} E_1 + \sqrt{\frac{1-\delta}{2}} E_2 e^{i\phi} \tag{2}$$

where θ is the phase angle between the output electromagnetic wave at port 3 and the input electromagnetic wave at port 1, ϕ is the phase angle between the input electromagnetic waves at port 2 and port 1, and E is the magnitude of $\boldsymbol{E}$.

The relationship between the powers at ports 1, 2, and 3 is from (2)

$$P_3' \equiv P_\phi = \left(\sqrt{\frac{1+\delta}{2} P_1} + \sqrt{\frac{1-\delta}{2} P_2} e^{i\phi}\right) \left(\sqrt{\frac{1+\delta}{2} P_1} + \sqrt{\frac{1-\delta}{2} P_2} e^{-i\phi}\right) \tag{3}$$

where the term in the second parenthesis on the right-hand side of (3) is the complex conjugate of the term in the first parenthesis. Multiplication of the two terms on the right-hand side of (3) gives

$$P_3' = P_\phi = \tfrac{1}{2} \left[P_1(1+\delta) + P_2(1-\delta) + 2\sqrt{P_1 P_2 (1-\delta^2)} \cos\phi\right]. \tag{4}$$

Coupled mode analysis for the Y junction [2] can be extended to obtain the same result as (4) [3].

Let us first consider the perfect 3 dB coupler ($\delta = 0$) and note for this case that the scattering matrix (1) is of the same form as the scattering matrix of the "magic T" in microwave circuits and also that of the hybrid coil used in telephone repeater circuits, both of which are also characterized by no reflections. If one assumes that the powers incident on port 1 and port 2 are different—that the power into port 1 is $P_o(1+\Delta)$ and the power into port 2 is $P_o(1-\Delta)$—then as the phase between these input powers is varied over π, the maximum-to-minimum ratio of the output power P_3 is from (4)

$$\frac{P_{3\max}}{P_{3\min}} = \frac{(1+\sqrt{1-\Delta^2})^2}{\Delta^2}. \tag{5}$$

The ratio in (5) is plotted in Fig. 2. Note that for $\Delta = 0.1$, which is equivalent to $P_1/P_2 = 1.22$

$$\frac{P_{3\max}}{P_{3\min}} = 398 \text{ or } 26 \text{ dB}. \tag{6}$$

The fact that this ratio is still large for relatively large differences in the power in the two input arms has a significant impact on the operation of the interferometric modulator shown in Fig. 3. It means that the operation of this modulator is extremely tolerant to variations in the power division at the input near 3 dB coupler and/or unequal absorption of the power in the respective input guides both in their straight and their "bend" sections. If because of fabrication errors or other causes there is a 20 percent difference in the power in the two

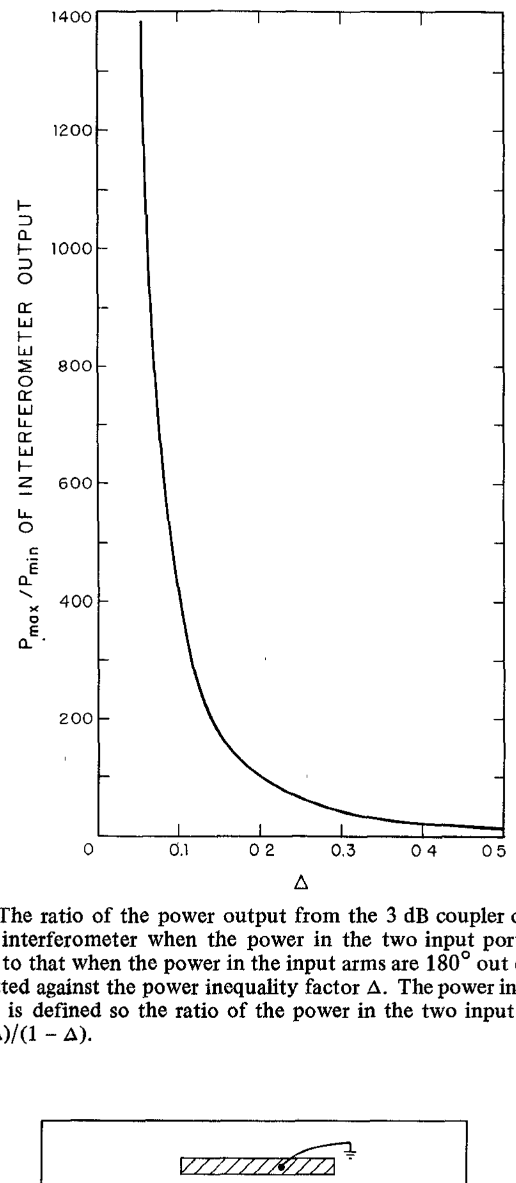


Fig. 2. The ratio of the power output from the 3 dB coupler operated as an interferometer when the power in the two input ports are in phase to that when the power in the input arms are 180° out of phase is plotted against the power inequality factor Δ. The power inequality factor is defined so the ratio of the power in the two input ports is (1 + Δ)/(1 - Δ).

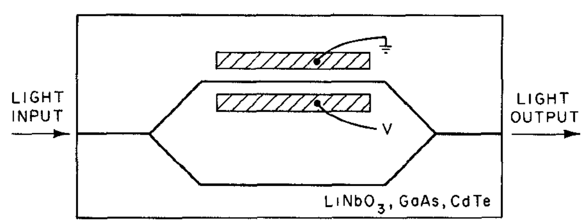


Fig. 3. The Mach-Zehnder interferometric modulator.

arms which feed the output Y junction, the peak-to-valley ratio of power output of the interferometric modulator will be larger than 26 dB! Conversely, large peak-to-valley ratios in the output of the interferometric modulator must not be interpreted in terms of having solved fabrication and/or design problems in making the input 3 dB coupler, the waveguides, and their bends.

For a *near* 3 dB coupler, as the output coupler of the interferometric modulator and as above assuming unequal powers in port 1 and port 2, i.e., $P_1 = P_o(1 + \Delta)$ and $P_2 = P_o(1 - \Delta)$, then as the phase between these input powers is varied over π, the maximum-to-minimum ratio of the interferometer output power is from (4)

$$\frac{P_{3\max}}{P_{3\min}} = \frac{\left(1 + \sqrt{(1 - \delta^2)(1 - \Delta^2)} + \delta\Delta\right)^2}{(\Delta + \delta)^2}. \tag{7}$$

In Fig. 4 the ratio of (7) is plotted as a function of the imbalance δ in the near 3 dB coupler for various values of the power inequality factor (Δ) in the two arms of the interferometer. Also shown in the figure as a dashed line is the case for $\Delta = \delta$. The insensitivity to the imbalance in the output coupler is important to the practical applicability of the interferometric modulator. For equal power into the two arms ($\Delta = 0$), a fabrication or design error that produced a $\delta = 0.1$ would yield

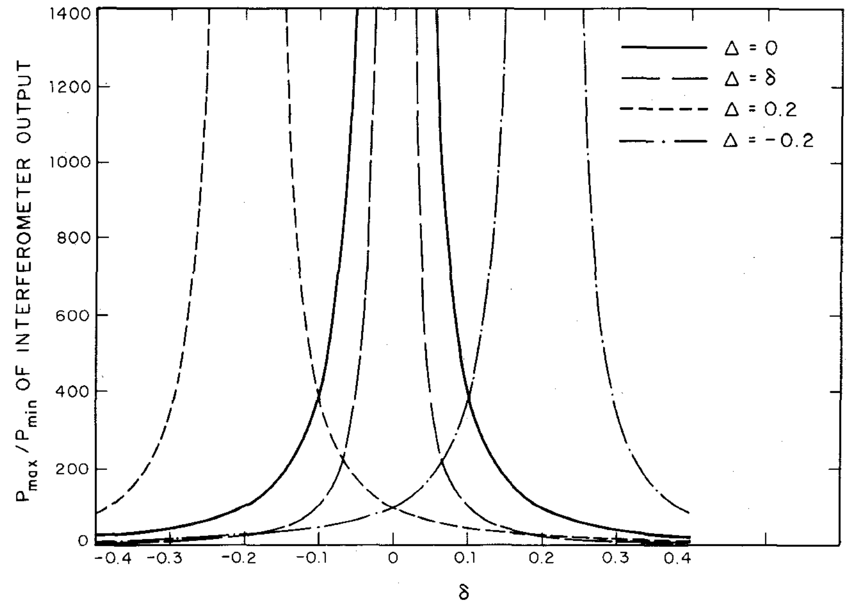


Fig. 4. The maximum-to-minimum ratio of the output of the Mach–Zehnder interferometric modulator as a function of the imbalance parameter δ of the output coupler with Δ as parameter. The quantity δ is defined in terms of this near 3 dB coupler being operated as a power divider, in which case it would couple $\frac{1}{2}(1 + \delta)$ of the input power into one arm and $\frac{1}{2}(1 - \delta)$ of the input power into the other arm.

a quite acceptable $P_{3\max}/P_{3\min}$ of 26 dB, while if the coupler were operated in the reverse direction, 22 percent more power would be coupled into one arm than the other, which most probably would be unacceptable for a power divider. In any practical case there would be both imbalance in the output 3 dB coupler and inequality in the power in the arms feeding it. If it is assumed that the errors are systematic and additive, the worst case is for dashed line in Fig. 4. Note that for a 10 percent imbalance and a 10 percent power inequality $\Delta = \delta = 0.05$ the ratio of $P_{3\max}/P_{3\min}$ has the quite acceptable value of 400 or 26 dB.

Conclusion

In conclusion, the operation of the Mach–Zehnder interferometric modulator has been shown to be relatively insensitive to the power inequality in the arms feeding the near 3 dB output coupler and also relatively insensitive to the imbalance in this coupler that would make the coupler's operation as a power divider very marginal.

The use of the scattering matrix formalism for a lossless four-port device has been shown to be a powerful tool in the analysis of the interferometric properties of the integrated-optics 3 dB coupler as exemplified by single-mode waveguide Y junction, just as it was over 35 years ago in the analysis of the microwave "magic T."

Acknowledgment

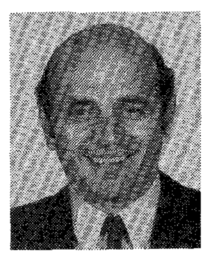
The authors thank R. H. Kingston for bringing the scattering matrix formalism to their attention and for continued helpful discussions.

References

[1] For a discussion of the scattering matrix formalism and an example of its use in microwave circuits, see C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits.* New York: McGraw-Hill, 1948, pp. 139–161.

[2] See, for example, W. K. Burns and A. F. Milton, "An active analytic solution for mode coupling in optical waveguide branches," *IEEE J. Quantum Electron.*, vol. QE-16, pp. 446–454, Apr. 1980.

[3] W. K. Burns, private communication.



Robert H. Rediker (S'47–A'53–SM'58–F'62) was born in Brooklyn, NY, on June 7, 1924. He received the B.S. degree in electrical engineering and the Ph.D. degree in physics, both from the Massachusetts Institute of Technology, Cambridge, in 1947 and 1950, respectively.

In 1951 he was employed by the Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, where he worked on semiconductor devices. He was one of the original workers in the field of GaAs devices and was a member of the team that developed the high-efficiency GaAs light-emitting diode and the GaAs laser diode. He was Professor of Electrical Engineering at M.I.T. from 1966 to 1976 and has been Adjunct Professor since 1976. His work on the M.I.T. campus has been centered on optoelectronic semiconductor devices. From 1970 to 1972 he was Associate Head and from 1972 to 1980 he was Head of the Optics Division, Lincoln Laboratory. In 1980 he returned to research at Lincoln Laboratory where his work has centered on guided-wave optoelectronic devices.

Dr. Rediker is a Fellow of the American Physical Society and a member of the Optical Society of America and Sigma Xi. He received the 1969 IEEE David Sarnoff Award.

Frederick J. Leonberger (M'75–SM'81) was born in Washington, DC, on September 25, 1947. He received the B.S.E. degree from the University of Michigan, Ann Arbor, in 1969, and the S.M., E.E., and Ph.D. degrees from the Massachusetts Institute of Technology, Cambridge, in 1971, 1972, and 1975, respectively, all in electrical engineering.

In 1975 he joined the Solid State Division, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, as a Staff Member. He has worked on GaAs and $LiNbO_3$ integrated optical devices, InP optoelectronic switches, ion implantation, and acoustoelectric imaging devices. He is currently Assistant Leader of the Applied Physics Group.

Dr. Leonberger is a member of Sigma Xi and Tau Beta Pi.